Int J Theor Phys (2008) 47: 2533–2542
DOI 10.1007/s10773-008-9687-0

Hawking Radiation from the Higher-Dimensional Schwarzschild de Sitter and Anti-de Sitter Black Holes via Covariant Anomaly

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Received: 30 November 2007 / Accepted: 8 February 2008 / Published online: 20 February 2008

Abstract Very recently, a conceptually clean and economical anomaly cancellation method, based on the initial work of Robinson and Wilczek, on Hawking radiation was proposed. On the basis of this formalism, we investigate Hawking radiation from the higher dimensional Schwarzschild de Sitter and Anti-de Sitter black holes. To describe the observable physics in de Sitter space, we construct the effective field theory between the event horizon and cosmological horizon. Our result shows that when the underlying diffeomorphism symmetries are saved at the quantum level, Hawking radiation, from not only the event horizon but also the cosmological horizon in the higher dimensional space time, can be determined by the covariant compensating fluxes of energy momentum tensor. Meanwhile, we also discuss the exact radiation spectrum by incorporating the self-gravitational interaction and back reaction of the outgoing modes.

Keywords Hawking radiation · Covariant anomaly · Cosmological horizon · Event horizon

1 Introduction

Since Bernhard Riemann suggested the universe maybe has more than three spaces firstly, the spatial dimensions have been discussed extensively [1, 2]. String theory predicts there could be up to $n = 7$ additional dimensions, compactified at distances of the order of 10^{-32} m. These large extra dimensions are believed may solve the hierarchy problem of the standard model [3]. In addition, since the introduction of extra dimensions would affect both gravitational interactions and particle physics phenomenology, it will also inevitably change our notion for the universe and even leads to modifications of the standard cosmology [4–7]. Preceding in importance, by lowering the Planck scale closer to the Electro-Weak scale in the higher dimensions, the idea of detection on Hawking radiation as an exciting possibility of finding new physics by making mini black holes in the Large Hadron Collider (LHC)

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next year [8] becomes more realistic. Thus investigation on higher dimensional space time is of great importance and necessary. In this paper, we study Hawking radiation from the higher dimensional Schwarzschild de Sitter and Anti-de Sitter black hole via the anomaly cancellation method. This approach, to some extent is motivated by the trance anomaly [9], is proposed recently by Robinson and Wilczek [10]. And the core of this is that scalar field on arbitrary space time can be reduced to that on (t, r) section of the original background. Then in the two dimensional reduction, due to the ingoing modes classically don't affect the outside physics, they hence integrate out these offending modes to construct the effective field theory outside the event horizon (EH). At the quantum level, the effective field becomes chiral and suffers from gravitational anomaly with respect to the general coordinate symmetry. The underlying theory of course should be covariant at the EH, thus they introduced a flux to cancel this nonexistent anomaly at the quantum level and eventually found this compensating flux equal to that of Hawking radiation exactly. Till now, based on their observations, investigations on higher dimensional space time [11–13], de-Sitter and anti-de-Sitter space time [14, 15] have been treated of. However, less attention paid on the higher dimensional black hole in de-Sitter and anti-de-Sitter space-time. Recent observations, together with the theory of inflation, suggest our universe is accelerating expansion [16, 17]; one of the possible explanations is the existence of a positive cosmological constant [18–21]. Thus it is of great significant and necessary to investigate the basic properties, such as Hawking radiation, thermodynamics, dS/CFT correspondence etc., of black holes in de Sitter space. Moreover, Anti-de Sitter black holes play an important role in the AdS/CFT conjecture [22, 23] and are interested in the context of brane world scenarios [24]. In light to above considerations, to further discover the quantum gravitational mysteries, as an intermediate, we study Hawking radiation from the higher dimensional Schwarzschild de Sitter and Anti-de Sitter black hole from the viewpoint of anomaly. It should be noted that in the de Sitter space, future infinity is space-like, to describe the observable physics, we construct the effective field between the EH and cosmological horizon (CH). Then, while we deal with the quantum anomaly at the EH, we ignore that at CH. In the same way, when the quantum anomaly at the CH is discussed, the quantum contribution of the ingoing modes at the EH are taken into account. It should pointed that in this paper, we don't adopt the initial approach of Robinson and Wilczek, which need both the consistent anomaly and covariant anomaly to determine the compensating flux, but only employ the covariant anomaly cancellation conditions. That is, one can determine the Hawking radiation flux only by adopting the covariant anomaly. Our result is precisely the same as that obtained by the others.

The remainder of this paper is organized as follows. In Sect. 2, we will introduce the higher dimensional Schwarzschild de Sitter black hole and study its quantum anomaly at the EH and CH. Then in Sect. 3, we turn to studying the quantum anomaly at the EH of the higher dimensional Schwarzschild Anti-de Sitter black hole. Section 4 is devoted to our concluding remarks, particularly, we discussion the exact emission spectrum by the consideration of energy conservation in the dynamical background space time.

2 Quantum Anomaly and Hawking Radiation of the Higher Dimensional Schwarzschild de Sitter Black Hole

The line element of the $(n+2)$-dimensional Schwarzschild de Sitter black hole with a positive cosmological constant $\Lambda = n(n+1)/2l^2$ takes the form as [25]

$$ds^2 = -f dt_d^2 + f^{-1} dr^2 + r^2 d\Omega_n^2, \tag{1}$$

where

$$f=\left(1-\frac{\omega_n M}{r^{n-1}}-\frac{r^2}{l^2}\right),\quad \omega_n=\frac{16\pi G_{n+2}}{nV_n},\tag{2}$$

V_n represent the volume of $d\Omega_n^2$, which is a n-dimensional spherical hyper-surface, G_{n+2} is the Newton's constant, M is the conserved mass of the black hole and l denote the curvature radius of the de-Sitter space. Obviously for $M=0$, line element (1) reduces to the pure de-Sitter space time. The quantum anomaly of the pure 4-dimensional de-Sitter space-time at the CH $r_c=\sqrt{3}/\sqrt{\Lambda}$ has been discussed detailed in reference [14]. According to the null hyper-surface equation, $f(M,r)=0$ will yield the CH, denoted by r_c and the EH, represented by r_+ respectively. The corresponding surface gravity at the EH and CH can be written as

$$\kappa_+=\frac{1}{2}\partial_r f\Big|_{r=r_+}=\frac{(n-1)\omega_n M}{2r_+^n}-\frac{r_+}{l^2},\tag{3}$$

$$\kappa_c=-\frac{1}{2}\partial_r f\Big|_{r=r_c}=-\left[\frac{(n-1)\omega_n M}{2r_c^n}-\frac{r_c}{l^2}\right].\tag{4}$$

Now, we focus on studying the quantum anomaly at horizon. As showed above, there are two observable horizons in the higher dimensional Schwarzschild de Sitter space time, namely the EH and CH. We first take into account the quantum effect of the outgoing modes at the CH to study the quantum anomaly at the EH. To do it, one should construct an effective field that requires reducing the higher dimensional space time to its (t,r) section of the original background. Considering matter fields, for the sake of simplicity only for scalar field, in the higher dimensional Schwarzschild de Sitter black hole. The action of scalar field φ is

$$\begin{aligned}S[\varphi]&=\frac{1}{2}\int d^{n+2}x\sqrt{-g}\varphi\nabla^2\varphi\\&=\frac{1}{2}\int d^{n+2}xr^n\sqrt{\gamma}\varphi\left[-\frac{1}{f}\partial_t^2+\frac{1}{r^n}\partial_r r^n f\partial_r+\frac{1}{r^2}\Delta_\Omega\right]\varphi,\end{aligned}\tag{5}$$

where γ and Δ_Ω respectively stands for the determinant of $d\Omega_n^2$ and the collection of angular derivatives. Based on the Legendre polynomial P_n, we expand the scalar field as $\varphi=\sum_n\phi_\omega(t,r)P_n$ and perform the tortoise coordinate transformation defined by $dr_*/dr=f(r)^{-1}$, after taking the near-horizon limit and ignoring the subordinate terms, we find

$$S[\varphi]=\sum_n\int\Psi dt dr\varphi_\omega\left[-\frac{1}{f(r)}\partial_t^2+\partial_r f(r)\partial_r\right]\varphi_\omega.\tag{6}$$

When the static dilaton background $\Psi=r_H^n/2$ is omitted [10], it straightforward shows the action of each partial wave modes on the higher dimensional Schwarzschild de Sitter black hole can be described effectively by an infinite collection of free, massless $(1+1)$-dimensional scalar field on the metric

$$ds^2=-f dt_d^2+f^{-1}dr^2.\tag{7}$$

In the 2-dimensional space time, since the EH is a null hyper-surface, when the classically irrelevant ingoing modes are excluded to construct an effective field outside the EH, due to the pileup of outgoing high frequency modes, the effective field becomes chiral and exhibits gravitational anomaly which fails to conserve the energy-momentum tensor. For the right handed field, the minimal form of the $(1+1)$-dimensional consistent anomaly reads [26, 27]

$$\nabla_\mu T^\mu_\nu = A_\nu = \frac{1}{\sqrt{-g}}\partial_\mu N^\mu_\nu, \tag{8}$$

where

$$N^\mu_\nu = \frac{1}{96\pi}\varepsilon^{\beta\mu}\partial_\alpha\Gamma^\alpha_{\nu\beta}, \tag{9}$$

$\varepsilon^{\mu\nu}$ is the anti-symmetric tensor ($\varepsilon^{10} = \varepsilon^{01} = -1$) and $\Gamma^\alpha_{\nu\beta}$ is the Christoffel connection on the $(1+1)$-dimensional space time. The covariant anomaly, on the other hand, also takes the form as [26, 27]

$$\nabla_\mu \tilde{T}^\mu_\nu = \frac{-1}{96\pi\sqrt{-g}}g_{\nu\alpha}\varepsilon^{\beta\alpha}\partial_\beta R = \frac{1}{\sqrt{-g}}\partial_\mu\tilde{N}^\mu_\nu. \tag{10}$$

For the 2-dimensional space-time, one gets

$$\tilde{N}^r_t = [2ff'' - (f')^2]/192\pi. \tag{11}$$

To determine the compensating flux, if we follow the initial approaches as in reference [10], one should express the variation of outgoing modes' action by the consistent anomaly, and then impose the covariant energy-momentum tensor to vanish at the EH. That is, one should employ two different anomalies to determine the compensating flux. Very recently, it was suggested [28] the same work can be done only by adopting the covariant anomaly. This approach is believed conceptually clean and economical. So thereafter we employ the simplified approach of canceling the covariant anomaly to investigate the quantum anomaly at horizon.

In the interest of simplicity, besides split the region outside the EH into $[r_+, r_+ + \varepsilon]$ and $[r_+ + \varepsilon, \infty]$, we introduce the scalar step function $\Theta_+ = \Theta(r - r_+ - \varepsilon)$ and scalar top hat function $H = 1 - \Theta_+$ to write the covariant energy-momentum tensor there as the sum of two regions, namely $\tilde{T}^\mu_\nu = \tilde{T}^u_{\nu(0)}\Theta_+(r) + \tilde{T}^\mu_{\nu(H)}H(r)$. When the classically insignificant ingoing modes are omitted, the covariant energy-momentum tensor is conserved in the region $[r_+ + \varepsilon, \infty]$, which relates

$$\nabla_\mu \tilde{T}^\mu_{\nu(0)} = 0. \tag{12}$$

While in $[r_+, r_+ + \varepsilon]$, due to the pileup of outgoing high frequency modes, the effective field becomes chiral and the diffeomorphism invariance is violated. Thus the covariant energy-momentum tensor is lo longer conserved but suffers from gravitational anomaly, which satisfies the covariant anomalous equation

$$\nabla_\mu \tilde{T}^\mu_{\nu(H)} = \frac{1}{\sqrt{-g}}\partial_\mu\tilde{N}^\mu_\nu. \tag{13}$$

Solving (12) and (13) for the $\nu = t$ component, we find

$$\tilde{T}^r_{(0)t} = e_0, \tag{14}$$

$$\tilde{T}^r_{(H)t} = e_H + \tilde{N}^r_t(r) - \tilde{N}^r_t(r_+), \tag{15}$$

where e_0 and e_H are two integration constants, which respectively stand for the covariant flux of energy-momentum tensor at infinity and the EH. Then (10) can be expressed as

$$\nabla_\mu \tilde{T}^\mu_t = \partial_r \tilde{T}^r_t = \partial_r[\tilde{N}^r_t H(r)] + (\tilde{T}^r_{(0)t} - \tilde{T}^r_{(H)t} + \tilde{N}^r_t)\delta(r - r_+ - \varepsilon). \tag{16}$$

In which, the first term has to be canceled by the quantum effects of the classically irrelevant ingoing modes. That is to say, there existence a Wess-Zumino term that would modify the covariant energy-momentum tensor $\tilde{T}^\mu$ as an anomaly-free one $\tilde{T}'^r_t = \tilde{T}^r_t - \tilde{N}^r_t H(r)$. Therefore to save the general covariance, the coefficient of the delta function should satisfies

$$e_0 = e_H - \tilde{N}^r_t(r_+). \tag{17}$$

When the integration constant $e_H = 0$ is determined by imposing the regularity condition that requires the covariant energy-momentum tensor to vanish at the EH, the total covariant compensating flux of energy-momentum tensor can be written as

$$e_0 = -\tilde{N}^r_t(r_+) = \frac{\pi T_+^2}{12}, \tag{18}$$

where

$$T_+ = \frac{\kappa_+}{2\pi} = \frac{f'(r)|_{r_+}}{4\pi} = \frac{1}{4\pi}\left[\frac{(n-1)\omega_n M}{r_+^n} - \frac{2r_+}{l^2}\right], \tag{19}$$

is the Hawking temperature at the EH of the higher dimensional Schwarzschild de Sitter black hole. Next we will show the blackbody radiation with this temperature is equivalent to the compensating flux of energy-momentum tensor precisely.

Hereinbefore, we taken into account the quantum effect of the outgoing modes at the CH to discuss the quantum anomaly at the EH and found the compensating flux of energy-momentum tensor can cancel the gravitational anomaly at the EH to hold the general covariance at the quantum level. Next, we will ignore the quantum anomaly at the EH to discuss Hawking radiation from the CH via the covariant anomaly cancellation method. Note that in de-Sitter space, future infinity is space-like, which means any physics are only taken place inside the CH. Thus, the effective field is constructed inside the CH when the classically irrelevant outgoing modes are integrated out. Quantum mechanically however, the contribution of outgoing modes at the CH cannot be neglected. Thus the effective field theory becomes chiral and suffers from gravitational anomaly with respect to the general coordinate symmetry.

In terms of the scalar step function $\Theta_- = \Theta(r_c - r - \varepsilon)$ and top hat function $C = 1 - \Theta_-$, we write the covariant flux of energy-momentum tensor inside the CH as $\tilde{T}^\mu_\nu = \tilde{T}^u_{\nu(0)}\Theta_-(r) + \tilde{T}^\mu_{\nu(c)}C(r)$, where $\tilde{T}^u_{\nu(0)}$ and $\tilde{T}^\mu_{\nu(c)}$ are the covariant fluxes in the region $r_c - \varepsilon \le r \le r_c$ and $r \le r_c - \varepsilon$. When the offending modes are ignored, they respectively satisfies

$$\nabla_\mu \tilde{T}^\mu_{\nu(0)} = 0, \tag{20}$$

$$\nabla_\mu \tilde{T}^\mu_{\nu(c)} = \frac{-1}{\sqrt{-g}}\partial_\mu \tilde{N}^\mu_\nu. \tag{21}$$

Solving them for the $\nu = t$ component in each region, we obtain

$$\tilde{T}^r_{(0)t} = c_0, \tag{22}$$

$$\tilde{T}^r_{(c)t} = c_c + N^r_t(r_c) - \tilde{N}^r_t(r). \tag{23}$$

Here c_c and c_0 are integration constants and c_0 represent the covariant flux of energy-momentum tensor at infinity. Correspondingly, the Ward identity inside the CH is

$$\nabla_\mu \tilde{T}^\mu_t = \partial_r \tilde{T}^r_t = -\partial_r[\tilde{N}^r_t C(r)] + (\tilde{T}^r_{t(c)} - \tilde{T}^r_{t(0)} + \tilde{N}^r_t)\delta(r - r_c + \varepsilon). \tag{24}$$

While the first term is canceled by the quantum effects of the classically irrelevant outgoing modes, which contribute to the total energy-momentum tensor $\tilde{N}^r_t C(r)$, the underlying general covariant conditions at the CH will impose the coefficient of the delta function follow

$$c_0 = c_c + \tilde{N}^r_t(r_c), \tag{25}$$

where, we have employed (22) and (23). To further get the observable covariant flux of energy-momentum tensor, we should determine c_c firstly. Considering the covariant boundary condition at the CH, from (23), we get $c_c = 0$. Thus the total compensating flux of energy-momentum tensor at the CH is

$$c_0 = \tilde{N}^r_t(r_c) = -\frac{\pi T_c^2}{12}. \tag{26}$$

In which, the Hawking temperature from the CH of the higher dimensional Schwarzschild de Sitter black hole have been expressed as

$$T_c = \frac{\kappa_c}{2\pi} = -\frac{f'(r)|_{r_c}}{4\pi} = -\frac{1}{4\pi}\left[\frac{(n-1)\omega_n M}{r_c^n} - \frac{2r_c}{l^2}\right]. \tag{27}$$

For the negative sign, it means to hold the underlying general coordinate invariance at the quantum level; the effective field should absorb the Hawking radiation flux from the CH. Now we explore the relation between the compensating flux and Hawking radiation flux. At the EH, the Planckian distributions of fermions with appropriate chemical potential is $N_+(\omega) = [\exp(\omega/T_+) + 1]^{-1}$. While at the CH, due to the motion of ingoing modes are along the negative r direction, the Planckian distributions of fermions thus takes the form as $N_c(\omega) = -[\exp(\omega/T_c) + 1]^{-1}$. Then we find

$$F_h = \int_0^\infty \frac{\omega}{\pi} N_+(\omega) d\omega = \frac{\pi T_+^2}{12}, \tag{28}$$

$$F_c = \int_0^\infty \frac{\omega}{\pi} N_c(\omega) d\omega = -\frac{\pi T_c^2}{12}. \tag{29}$$

Comparing the Hawking radiation fluxes in (28) and (29) with the compensating fluxes in (18) and (26), which can save the general covariance by canceling the gravitational anomaly at the EH and the CH respectively, we find they are equivalence precisely. That is, when the underlying general covariance are restored at the quantum level, Hawking radiation from the EH and CH of the higher dimensional Schwarzschild de Sitter black hole can be determined by the compensating fluxes. Our result is exactly the same as that calculated by cancelling the consistent one.

3 Quantum Anomaly and Hawking Radiation of the Higher Dimensional Schwarzschild Anti-de Sitter Black Hole

The line element of the $(d+1)$-dimensional Schwarzschild Anti-de Sitter black hole in the asymptotic coordinate takes the form as [29]

$$ds^2 = -Fdt_A^2 + F^{-1}dr^2 + r^2 d\Omega_{d-1}^2, \tag{30}$$

where

$$F = 1 - \frac{M\mu}{r^{d-2}} + r^2. \tag{31}$$

Note that here we have set the Anti-de Sitter radius $l = 1$. The parameter μ relates to the ADM mass as

$$\mu = \frac{16\pi G_{d+1}}{(d-1)A_{d-1}}. \tag{32}$$

In which, $A_{d-1} = 2\pi^{d/2}/\Gamma(d/2)$ denote the volume of a unit $(d-1)$-sphere and G_{d+1} represent the Newton's constant in the $(d+1)$-dimensional space. In addition, the explicit form of the horizon can be yielded from $F(r_+) = 0$, the surface gravity at the EH thus is

$$\kappa = \frac{1}{2}\partial_r F\Big|_{r=r_+} = \frac{1}{2}\left[\frac{(d-2)M\mu}{r_+^{d-1}} + 2r_+\right]. \tag{33}$$

Adopting similar dimensional reduction technique as above, the higher dimensional Schwarzschild Anti-de Sitter black hole can be effectively reduced as

$$ds^2 = -Fdt_A^2 + F^{-1}dr^2. \tag{34}$$

In the reduced space time, since the EH is a null hyper-surface, any observable physics only take place at the outside of the EH, hence we ignore the classically insignificant ingoing modes to formulate the effective field outside the EH. At the quantum level however, the contribution of ingoing modes can't be omitted. Thus under the diffeomorphism transformation, the general coordinate symmetry is breakdown which results in the emergence of gravitational anomaly. The underlying theory of course is covariant, thus one should introduce a flux to cancel the anomaly quantum mechanically. As before, we split the region outside the EH into $[r_+, r_+ + \varepsilon]$ and $[r_+ + \varepsilon, \infty]$. In the near-horizon region $[r_+, r_+ + \varepsilon]$, when the classically insignificant ingoing modes are omitted, due to the divergent energies of outgoing modes near-horizon modes, the covariant energy-momentum tensor suffers from gravitational anomaly which brings on

$$\nabla_\mu \tilde{T}^\mu_{\nu(H)} = \frac{1}{\sqrt{-g}}\partial_\mu \tilde{N}^\mu_\nu. \tag{35}$$

While in $[r_+ + \varepsilon, \infty]$, its partner follows

$$\nabla_\mu \tilde{T}^\mu_{\nu(0)} = 0, \tag{36}$$

In terms of the scalar step function and scalar top hat function, the covariant energy-momentum tensor outside the EH can be expressed as

$$\tilde{T}^\mu_\nu = \tilde{T}^u_{\nu(0)}\Theta_+(r) + \tilde{T}^\mu_{\nu(H)}H(r). \tag{37}$$

Here the $\nu = t$ component of $\tilde{T}^{u}_{\nu(0)}$ and $\tilde{T}^{\mu}_{\nu(H)}$ can be solves as

$$\tilde{T}^{r}_{(0)t} = a_0, \tag{38}$$

$$\tilde{T}^{r}_{(H)t} = a_H + \tilde{N}^r_t(r) - \tilde{N}^r_t(r_+). \tag{39}$$

So the Ward identity outside the EH is

$$\nabla_\mu \tilde{T}^\mu_t = \partial_r \tilde{T}^r_t = \partial_r[\tilde{N}^r_t H(r)] + (\tilde{T}^r_{(0)t} - \tilde{T}^r_{(H)t} + \tilde{N}^r_t)\delta(r - r_+ - \varepsilon). \tag{40}$$

Considering the Wess-Zumino term, one can write the covariant energy-momentum tensor as the anomaly-free one $\tilde{T}'^r_t = \tilde{T}^r_t - \tilde{N}^r_t H(r)$, that is, the first term will be canceled by the quantum effects of the classically irrelevant ingoing modes. Therefore, while the underlying general covariance is restored, the properties of the delta function would impose

$$a_0 = a_H - \tilde{N}^r_t(r_+), \tag{41}$$

where we have employed (38) and (39). To ensure the regularity condition at the CH, the covariant flux will vanish there, which leads to $a_H = 0$. The total covariant flux of energy-momentum tensor, which can cancel the gravitational anomaly at the EH to save the general coordinate covariance at the quantum level, thus is

$$a_0 = -\tilde{N}^r_t(r_+) = \frac{\pi T_+^2}{12}, \tag{42}$$

where

$$T_+ = \frac{\kappa_+}{2\pi} = \frac{f'(r)|_{r_+}}{4\pi} = \frac{1}{4\pi}\left[\frac{(d-2)M\mu}{r_+^{d-1}} + 2r_+\right] \tag{43}$$

is the Hawking temperature of the higher dimensional Schwarzschild Anti-de Sitter black hole. For the fermions, the Planckian distributions with this temperature take the form as $N(\omega) = [\exp(\omega/T_+) + 1]^{-1}$, and we find

$$F_a = \int_0^\infty \frac{\omega}{\pi} N(\omega) d\omega = \frac{\pi T_+^2}{12}. \tag{44}$$

That is, when the underlying general coordinate invariance are saved at the quantum level, Hawking radiation from the EH of the higher dimensional Schwarzschild Anti-de Sitter black hole can be determined by the compensating flux.

4 Concluding Remarks

We have derived Hawking radiation from the higher dimensional Schwarzschild de Sitter and Anti-de Sitter black holes. Both of them share a common that the horizons are null hyper-surfaces, when the classically irrelevant ingoing modes at the EH and outgoing modes at the CH are integrated out to construct the effective field theory, anomaly, the conflict between the conservation law and the quantization procedure, emerged. To cancel the nonexist anomaly at the quantum level, we introduced a covariant flux. And eventually we found the Hawking radiation fluxes, not only from the de Sitter space but also from the Anti-de Sitter space, equal to the introduced fluxes exactly. Our result agrees with the initial viewpoint of Robinson and Wilczek that Hawking radiation can be determined by the compensating fluxes at the horizon.

But notably, the obtained Hawking radiation spectrums there are only the pure thermal one. In fact, Parikh and Wiclzek [30–32] several years ago have proved that the actual emission spectrum isn't pure via an intuitively simple but physically rich semi-classical approach. They meanwhile pointed out the correction is the response of the self-gravitational interaction and back reaction of the radiation. Therefore to get the actual radiation spectrum, we should take the energy conservation into account. We now take the higher dimensional Schwarzschild Anti-de Sitter black hole as an example. When the particle with a shell of energy ω runs out, the radius of the horizon shall shrink and become as the function of $M-\omega$. In this case, the imaginary part of the radiation's action takes the form as

$$\mathrm{Im}\, I = \frac{2\pi r_f^{d-2}\omega}{(d-2)(M-\omega)\mu + 2r_f^{d-1}}, \tag{45}$$

where r_f satisfies

$$1 - \frac{M\mu}{r_f^{d-2}} + r_f^2 = 0. \tag{46}$$

Due to the restriction of the quantum uncertainty principle, the black hole mass can't jump in such a discontinuous manner from M to $M-\omega$ after a particle runs out. Thus we integrate over ω in the region $0 \to \omega$. That is

$$\mathrm{Im}\, I' = \int_0^{\omega} \frac{2\pi r_f^{d-2} d\omega}{(d-2)(M-\omega)\mu + 2r_f^{d-1}}. \tag{47}$$

Finishing the integration, we obtain

$$\mathrm{Im}\, I' = \frac{2\pi}{\mu(d-1)}[r_+^{d-1}(M) - r_f^{d-1}(M-\omega)]. \tag{48}$$

Employing the semi-classical WKB approximation, the emission rate hence can be expressed explicitly as

$$\Gamma \sim \exp(-2\mathrm{Im}\, I) = \exp\left\{\frac{A_{d-1}}{4\pi G_{d+1}}[r_f^{d-1}(M-\omega) - r_+^{d-1}(M)]\right\} = \exp(\Delta S_{BH}), \tag{49}$$

where $\Delta S_{BH} = S(r_f) - S(r_+)$ is the change of Bekenstein–Hawking entropy of the black hole that after and before the radiation. This result is consistent with the initial viewpoint of Parikh and Wilczek that the tunneling rate is related to the change of Bekenstein–Hawking entropy.

Thus, as the self-gravitational interaction and back reaction of the radiation is considered, the thermal spectrum of the outgoing modes deviate from the pure thermal one. By the similar way of course, we also can find the correction spectrum of the higher dimensional Schwarzschild de Sitter black hole.

Acknowledgements This work is supported by the National Natural Science Foundation of China under Grant No. 10773008.

References

1. Arkani-Hamed, N., Dimopoulos, S., Dvali, G.: Phys. Lett. B **429**, 263 (1998)
2. Stojkovic, D.: Phys. Rev. Lett. **94**, 011603 (2005)
3. Landsberg, G.: Phys. Rev. Lett. **88**, 181801 (2002)
4. Green, A.M., Lidsey, J.E.: Phys. Rev. D **61**, 067301 (2000)
5. Hewett, J.L.: Phys. Rev. Lett. **82**, 4765 (1999)
6. Han, T., Lykken, J.D., Zhang, R.J.: Phys. Rev. **59**, 105006 (1999)
7. Kaloper, N., Liddle, A.R.: Phys. Rev. D **61**, 123513 (2000)
8. Giddings, S.B.: arXiv:0709.1107[hep-th] (2007)
9. Christensen, S., Fulling, S.: Phys. Rev. D **15**, 2088 (1977)
10. Robinson, S.P., Wilczek, F.: Phys. Rev. Lett. **95**, 011303 (2005)
11. Iso, S., Morita, T., Umetsu, H.: J. High Energy Phys. **0704**, 068 (2007)
12. Shin, H., Kim, W.: High Energy Phys. **06**, 012 (2007)
13. Xu, Z., Chen, B.: Phys. Rev. D **75**, 02404 (2007)
14. Jiang, Q.Q.: Class. Quantum Gravity **24**, 4391 (2007)
15. Jiang, Q.Q., Wu, S.Q., Cai, X.: Phys. Lett. B **647**, 200 (2007)
16. Ostriker, J.P., Steinhardt, P.J.: Nature **377**, 600 (1995)
17. Bahcall, N., Ostriker, J.P., Perlmutter, S., Steinhardt, P.J.: Science **284**, 1481 (1999)
18. Perlmutter, S., et al.: Astrophys. J. **483**, 565 (1997)
19. Riess, A.G., et al.: Astron. J. **116**, 1009 (1998)
20. Moffat, J.W.: AIP Conf. Proc. **646**, 130 (2003)
21. Perlmutter, S., et al.: Astron. J. **51**, 565 (1999)
22. Maldacena, J.: Adv. Theor. Math. Phys. **2**, 231 (1998)
23. Witten, E.: Adv. Theor. Math. Phys. **2**, 253 (1998)
24. Randall, L., Sundrum, R.: Phys. Rev. Lett. **83**, 3370 (1999)
25. Medved, A.J.M.: Phys. Rev. D **66**, 124009 (2002)
26. Bertlmann, R., Kohlprath, E.: Ann. Phys. (NY) **288**, 137 (2001)
27. Bardeen, W.A., Zumino, B.: Nucl. Phys. B **244**, 421 (1984)
28. Banerjee, R., Kulkarni, S.: arXiv:0707.2449 [hep-th] (2007)
29. Hemming, S., Keski-Vakkuri, E.: Phys. Rev. D **64**, 44006 (2001)
30. Parikh, M.K., Wilczek, F.: Phys. Rev. Lett. **85**, 5042 (2000)
31. Zhang, J.Y., Zhao, Z.: Mod. Phys. Lett. A **20**, 1673 (2005)
32. Yang, S.Z.: Chin. Phys. Lett. **22**, 2492 (2005)